ONE-DIMENSIONAL MOTION OF AN EMULSION WITH SOLIDIFICATION

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A mathematical model is proposed for the process of solidification of an emulsion with a small disperse-phase concentration moving under the action of thermocapillary forces and microgravity. The first-approximation problem that arises when solutions are represented as asymptotic series in a small parameter is examined. Conditions for the partial and complete displacement of the impurity from the solidified part and conditions for the accumulation of the impurity in the solidified mixture are obtained. The problem of producing a composite with a specified disperse-phase distribution is considered. Exact solutions that adequately reflect various features of the qualitative behavior of the general solution under different input data are obtained and examined.

1. FORMULATION OF THE PROBLEM

1.1. Basic Assumptions. (1) The one-dimensional motion of an emulsion under the action of thermocapillary forces and microgravity is described by the mathematical model proposed in [1]. The required quantities are the temperature T and concentration of the disperse phase c. After they are determined, the rate and pressure are obtained from additional equations.

2. The process of solidification is described within the framework of the classical Stefan problem ignoring the jump in density during solidification.

3. The solidified matrix is immovable, and hence, the volume average rate of motion of the mixture is equal to zero.

Under the above assumptions, the temperature of the mixture and the concentration of the disperse phase of the liquid mixture are defined by the equations

$$\frac{\partial}{\partial t}c + \frac{\partial}{\partial x}\left(c(1-c)\left(L\frac{\partial T}{\partial x} + Kg\right)\right) = 0,$$

$$\rho_d\lambda_d c\left\{\frac{\partial T}{\partial t} + (1-c)\left(L\frac{\partial T}{\partial x} + Kg\right)\frac{\partial T}{\partial x}\right\} + \rho_m\lambda_m(1-c)\left\{\frac{\partial T}{\partial t} - c\left(L\frac{\partial T}{\partial x} + Kg\right)\frac{\partial T}{\partial x}\right\}$$

$$= k_m\frac{\partial}{\partial x}\left((1-Mc)\frac{\partial T}{\partial x}\right). \tag{1.1}$$

Here $M = 3(k_m - k_d)/(2k_m + k_d)$, $K = 2R^2(\rho_d - \rho_m)(\mu_d + \mu_m)/(3\mu_m(2\mu_m + 3\mu_d))$, and $L = 2Rk_m\sigma\mu_m/((2\mu_m + 3\mu_d)(2k_m + k_d))$, where ρ_d, ρ_m, λ_d, λ_m, μ_d, μ_m, k_d, and k_m denote the density, heat capacity, dynamic viscosity, and thermal conductivity in the disperse (subscript d) and carrier (subscript m) phases (these coefficients are considered positive and constant), R is the radius of disperse particles, σ is a positive constant that is the coefficient of the linear dependence of surface tension on temperature, and g is the constant acceleration of

microgravity, which is considered positive if the acceleration of gravity is directed toward higher temperature; otherwise, it is negative.

In the liquid matrix with solidified inclusions, there is no thermocapillary effect, and, therefore, in Eq. (1.1), one should set $L = 0$. Finally, in the solid matrix there are neither the thermocapillary effect nor Archimedes force, and, hence, $L = Kg = 0$. The conditions on a line of strong discontinuity have the form

$$[c]D = [c(Kg + LT_x)(1 - c)], \qquad [T] = 0,$$

$$[U]D = [(\rho_d\lambda_d - \rho_m\lambda_m)c(1 - c)(Kg + LT_x)T] - [k_m(1 - Mc)T_x],$$

(1.2)

where [.] denotes the difference between the function values on different sides of the discontinuity line, the jump in internal energy $[U]$ is considered constant and equal to γ, and D is the velocity of the phase boundary.

We introduce the "reduced concentration" $C = c/\varepsilon$, where ε is the maximum value of the initial concentration ($\varepsilon \ll 1$). The new function will be called by the former name, and its values are no longer restricted from above by unity.

Equations (1.1) and (1.2) become

$$\frac{\partial}{\partial t}C + \frac{\partial}{\partial x}\left(C(1 - \varepsilon C)\left(L\frac{\partial T}{\partial x} + Kg\right)\right) = 0,$$

$$\rho_d\lambda_d\varepsilon C\left\{\frac{\partial T}{\partial t} + (1 - \varepsilon C)\left(L\frac{\partial T}{\partial x} + Kg\right)\frac{\partial T}{\partial x}\right\} + \rho_m\lambda_m(1 - \varepsilon C)\left\{\frac{\partial T}{\partial t} - \varepsilon C\left(L\frac{\partial T}{\partial x} + Kg\right)\frac{\partial T}{\partial x}\right\}$$

$$= k_m\frac{\partial}{\partial x}\left((1 - M\varepsilon C)\frac{\partial T}{\partial x}\right),$$

and the strong discontinuity conditions are

$$[C]D = [C(Kg + LT_x)(1 - \varepsilon C)], \qquad [T] = 0,$$

$$[U]D = [(\rho_d\lambda_d - \rho_m\lambda_m)\varepsilon C(1 - \varepsilon C)(Kg + LT_x)T] - [k_m(1 - M\varepsilon C)T_x].$$

1.2. Additional Assumptions. The assumption of low disperse-phase concentration gives ground for linearization, which is the main method of studying the problem in the present work. Since thermal conduction as a first approximation ($\varepsilon = 0$) is determined by the parameters of the carrier phase (matrix), we assume that:

— The boundary of solidification of the matrix $x = s_m(t)$ is a Stefan boundary for the thermal problem, and the conditions of strong discontinuity for the temperature and concentration are satisfied on the boundary;

— The isotherm $T = T^d$, where T^d is the solidification temperature of the disperse phase, can be a strong discontinuity line only for the disperse-phase concentration [$x = s_d(t)$ is treated as the equation of this isotherm];

— The heat flux is directed toward increasing the x coordinate.

Let the solidification temperature of the matrix T^m be lower than the solidification temperature of the disperse phase T^d. The concentration of the solid disperse phase in the solidified matrix [the region $x < s_m(t)$] is denoted by $C^s(x,t)$, the concentration of the solid disperse phase in the liquid matrix (the region $s_m < x < s_d$) is denoted by $C^{sl}(x,t)$, and, finally, the concentration of the liquid disperse phase in the liquid matrix [the region $x > s_d(t)$] is denoted by $C^l(x,t)$. As a result of expansion in the small parameter ε, the problem of determining the functions of the first approximation $C^{sl}(x,t)$, $C^l(x,t)$, and $C^{sl}(x,t)$ takes the form

$$\frac{\partial}{\partial t}C^l + \frac{\partial}{\partial x}\left(C^l\left(L\frac{\partial T^l}{\partial x} + Kg\right)\right) = 0, \qquad x > s_d(t),$$

$$\frac{\partial}{\partial t}C^{sl} + \frac{\partial}{\partial x}(C^{sl}Kg) = 0, \qquad s_m(t) < x < s_d(t),$$

$$\frac{\partial}{\partial t} C^s = 0, \quad x < s_m(t), \quad \frac{ds_d}{dt}(C^l - C^{sl}) = C^l\left(L\frac{\partial T^l}{\partial x} + Kg\right) - C^{sl}Kg, \qquad x = s_d(t), \tag{1.3}$$

$$\frac{ds_m}{dt}(C^{sl} - C^s) = C^{sl}Kg, \qquad x = s_m(t).$$

The temperature $T^l(x,t)$ in the liquid mixture and the position of the free boundary $s_m(t)$ are tentatively obtained by solving the classical Stefan two-phase problem

$$\rho_m^l\lambda_m^l\frac{\partial T^l}{\partial t} = k_m^l\frac{\partial^2 T^l}{\partial x^2}, \quad x > s_m(t), \quad \rho_m^s\lambda_m^s\frac{\partial T^s}{\partial t} = k_m^s\frac{\partial^2 T^s}{\partial x^2}, \quad x < s_m(t),$$

$$T^l = T^s = T^m, \qquad \frac{ds}{dt}\gamma = k_m^s\frac{\partial T^s}{\partial x} - k_m^l\frac{\partial T^l}{\partial x}, \qquad x = s_m(t) \tag{1.4}$$

subject to certain boundary and initial conditions, for example,

$$T^s(0,t) = f^s(t) < T^m, \quad T_x^l(\infty,t) = 0, \quad T^s(x,0) = \varphi^s(x) < T^m, \quad x \in (0, s(t)),$$

$$T^l(x,0) = \varphi^l(x) > T^m, \quad x \in (s(t),\infty), \quad s(0) = s_0. \tag{1.5}$$

The boundary $x = s_d(t)$ is defined as the isotherm $T^l(s_d(t),t) = T^d$.

We now assume that the solidification temperature of the matrix T^m is higher than the solidification temperature of the disperse phase T^d. Since, in the solidified matrix, the disperse inclusions are immovable even if they have not solidified, the solidification boundary of the disperse phase $x = s_d(t)$ is no longer the line of concentration discontinuity. The problem of determining the disperse-phase concentration in the solidified matrix $C^s(x,t)$ and the disperse-phase concentration in the liquid matrix $C^l(x,t)$ consists of the equations

$$\frac{\partial}{\partial t} C^l + \frac{\partial}{\partial x}\left(C^l\left(L\frac{\partial T^l}{\partial x} + Kg\right)\right) = 0, \qquad x > s_m(t); \tag{1.6}$$

$$\frac{\partial}{\partial t} C^s = 0, \qquad x < s_m(t) \tag{1.7}$$

and the condition

$$\frac{ds_m}{dt}(C^l - C^s) = C^l\left(L\frac{\partial T^l}{\partial x} + Kg\right), \qquad x = s_m(t). \tag{1.8}$$

The functions $T^l(x,t)$ and $s_m(t)$ are also solutions of the classical two-phase Stefan problem (1.4), (1.5).

For the systems described, we formulate the following two problems: the problem of determining the concentration distribution in the solidified part from a specified initial distribution of the concentrations $C^{sl}(x,0) = C_0^{sl}(x) > 0$ and $C^l(x,0) = C_0^l(x) > 0$ in the liquid matrix (called a direct problem), and the problem of determining the initial concentration distribution in the liquid matrix from a specified concentration distribution $C_0^s(x) \geqslant 0$ in the solidified part (arbitrarily called an inverse problem). We search for a classical solution of the formulated problems with nonnegative restricted functions C^s C^{sl}, and C^l. For simplicity, it s assumed that the functions $C_0^l(x)$ and $C_0^s(x)$ are specified on semi-infinite intervals.

2. CONDITIONS OF PARTIAL AND COMPLETE DISPLACEMENT OF THE IMPURITY

We are interested in the process of "oriented solidification," i.e., solutions of the Stefan problem that satisfy the inequality $(ds_m/dt)(t) > 0$ for all values of time t. This requirement is easily ensured in terms of the input data of the Stefan problem [2]. In this case, by virtue of the maximum principle, $T_x^i(x,t) \geqslant 0$ and $T_{xx}^i(x,t) \leqslant 0$ $(i = s, l)$, and, hence, $(ds_d/dt)(t) \geqslant 0$ for all times t.

We consider the relation of the disperse-phase concentrations on different sides of the boundary of solidification of the matrix in the problem with one front (the case $T^m > T^d$). Taking condition (1.8) into
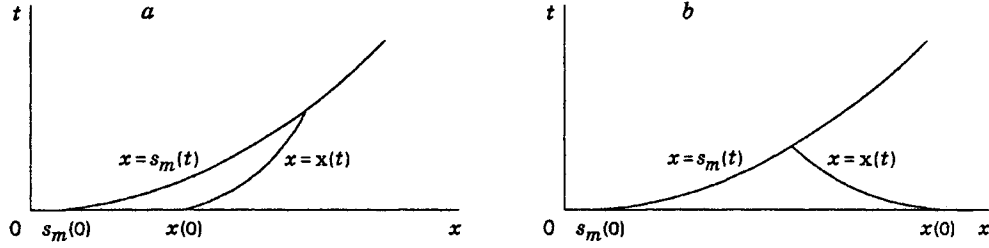
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account, we see that a necessary condition for the unique solvability of the direct and inverse problems is the satisfaction of the inequality

$$\frac{ds_m}{dt} \geqslant L\frac{\partial T^l}{\partial x}(s_m(t),t) + Kg, \qquad t > 0, \tag{2.1}$$

which ensures the "right" slope of the characteristics $x = \mathrm{x}(t)$ determined from the Cauchy problem

$$\frac{d\mathrm{x}}{d\tau} = L\frac{\partial T^l}{\partial x}(\mathrm{x}(\tau),\tau) + Kg, \qquad 0 \leqslant \tau \leqslant t, \qquad \mathrm{x}(t) = s_m(t), \tag{2.2}$$

and nonnegativity of the concentrations on the solidification boundary. Inequality (2.1) implies that the rate of transfer of the impurity in the liquid due to the thermocapillary effect and microgravity should not be higher than the rate of motion of the solidification boundary. Then, in the case

$$L\frac{\partial T^l}{\partial x}(s_m(t),t) + Kg \geqslant 0$$

(Fig. 1a), the inequality $C^s(x) \leqslant C^l(x, s_m^{-1}(x))$ holds, which implies that the impurity is displaced into the liquid phase. If

$$L\frac{\partial T^l}{\partial x}(s_m(t),t) + Kg < 0$$

(Fig. 1b), $C^s(x) > C^l(x, s_m^{-1}(x))$, i.e., the impurity is taken by the solidified part.

In the case of satisfaction of the identical equality

$$\frac{ds_m}{dt} = L\frac{\partial T^l}{\partial x}(s_m(t),t) + Kg,$$

the impurity is completely displaced from the solid phase, i.e., the direct problem can have only a trivial solution, and the inverse problem loses sense.

Finally, we consider the case

$$\frac{ds_m}{dt} < L\frac{\partial T^l}{\partial x}(s_m(t),t) + Kg, \qquad t > 0,$$

where the rate due to the thermocapillary effect and microgravity exceeds the rate of motion of the solidification boundary of the matrix. Then, by virtue of (1.8), in addition to $C^s(x) \equiv 0$, we have $C^l(s(t),t) \equiv 0$ for all times t.

We note that the conditions for the "right" slope of the characteristics for problem (1.3) have the form

$$\frac{ds_m}{dt} \geqslant Kg, \qquad \left(\frac{ds_d}{dt} - LT_x^l(s_d(t),t) - Kg\right)\left(\frac{ds_d}{dt} - Kg\right) \geqslant 0.$$

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3. SOLVABILITY OF THE PROBLEMS

Let us find sufficient conditions for the solvability of the direct and inverse problems for system (1.6)–(1.8).

Statement 3.1. *If*

$$-A\frac{ds_m}{dt}(t) \leqslant L\frac{\partial T^l}{\partial x}(s_m(t),t) + Kg \leqslant a\frac{ds_m}{dt}(t) \tag{3.1}$$

for all times t and some constants A and a that satisfy the conditions $A \geqslant 0$ and $0 \leqslant a < 1$, and there is a nonnegative constant N such that for all times t,

$$\left| \int_0^t \left(L\frac{\partial T^l}{\partial x}(s_m(\tau),\tau) + Kg \right)\frac{\partial T^l}{\partial x}(s_m(t)(1-a) + s_m(\tau)a,\tau)\,d\tau \right| \leqslant N,$$

then the solution $C^s(x)$ of problem (1.6)–(1.8) with the initial condition $C^l(x,0) = C_0^l(x) > 0$ $[x > s_m(0)]$ is nonnegative and bounded from above by

$$\max C_0^l(x)\exp\{Lk_m^{-1}((\varphi^l(x) - T^m) + N)\rho_m^l\lambda_m^l\}(1 + A).$$

Statement 3.2. *Let condition (3.1) be satisfied. Then, the solution $C^l(x,0) = C_0^l(x)$ of problem (1.5)–(1.8) with the condition $C^s(x,s_m^{-1}(x)) = C_0^s(x)$ is nonnegative and bounded from above by* $\max C_0^s(x)\exp\{Lk_m^{-1}N\rho_m^l\lambda_m^l\}/(1-a)$.

To prove Statements 1 and 2, we note that, by virtue of (1.8), we have

$$C^l(s_m(t),t)\left(\frac{d}{dt}s_m(t) - LT_x^l(s_m(t),t) - Kg\right) = C^s(s_m(t))\frac{d}{dt}s_m(t),$$

and by virtue of Eq. (1.6), we obtain

$$C^l(s_m(t),t) = C^l(x(0),0)\exp\left\{ -\int_0^t L\frac{\partial^2 T^l}{\partial x^2}(x(\tau),\tau)\,d\tau \right\},$$

where the characteristics $x(\tau)$ are determined from problem (2.2). Taking into account the equality

$$\frac{d}{d\tau}T^l(x(\tau),\tau) = \frac{\partial}{\partial x}T^l(x(\tau),\tau)\frac{\partial x}{\partial \tau} + \frac{\partial}{\partial \tau}T^l(x(\tau),\tau),$$

the first of Eqs. (1.4), the inequalities $T_x^l(x,t) \geqslant 0$ and $T_{xx}^l(x,t) \leqslant 0$, and the maximum principle for $C^l(x,t)$, we obtain what is required.

Remark 3.1. We consider the limiting case of the identical equality in condition (2.1), i.e., the situation where the line $s_m(t)$ is a characteristic. Then, in order that a solution of problem (1.6)–(1.8) with the initial conditions $C^l(x,0) = C_0^l(x) > 0$ $[x > s_m(0)]$ be bounded, it is necessary that the integral $\displaystyle\int_{s_m(0)}^{x} T_x^l(\xi,s_m^{-1}(\xi))\,d\xi$ be bounded uniformly on x. In this case, the impurity is completely displaced from the solid phase. The inverse problem in this case loses sense. We note that, this rather severe constraint is not satisfied for self-similar and traveling wave regimes of solidification.

As noted above, the condition of oriented solidification can be formulated in terms of the input data of the Stefan problem (1.4), (1.5). Comparison of the solution of the Stefan problem with an appropriate traveling wave solution makes it is possible to formulate sufficient (rather rough) constraints on the input data of the general problem under which the conditions of Statements 1 and 2 are satisfied, and both the direct and inverse problem have a classical solution. At the same time, the qualitative behavior of the solution of problem (1.6)–(1.8) is fairly well represented by the cases where the functions T^l and s_m are taken from a traveling wave solution or a self-similar (for $g = 0$) solution of the corresponding Stefan problem. We note that, in this case, the sufficient conditions of solvability formulated in Statements 1 and 2 are rather exact. We study these solutions.

4. SOME SPECIAL SOLUTIONS

4.1. Traveling Wave. Let $C^l(x,t) \to C_\infty$ as $x \to \infty$. Then, problem (1.4), (1.6)–(1.8) has a solution of the form

$$T^l(x,t) = A^l(1 - \exp\{-V\mathit{æ}^l(x - Vt)\}) + T^*,$$
$$T^s(x,t) = A^s(1 - \exp\{-V\mathit{æ}^s(x - Vt)\}) + T^*, \qquad s(t) = Vt; \tag{4.1}$$

$$C^l(x,t) = \frac{C_\infty(V - Kg)\exp\{V\mathit{æ}^l x\}}{(V - Kg)\exp\{V\mathit{æ}^l x\} - B\exp\{V^2\mathit{æ}^l t\}}, \qquad C^s(x,t) = C_\infty\left(1 - \frac{Kg}{V}\right), \tag{4.2}$$

where $\mathit{æ}^i = \rho_m^i \lambda_m^i / k_m^l$ $(i = s, l)$, $B = LVA^l\mathit{æ}^l$, the constants A^l and A^s are related by the Stefan condition from (1.4), V is an arbitrary positive constant which is not equal to Kg, and a nonnegative solution of problem (1.6)–(1.8) of this form exists if and only if $V \geqslant B + Kg$ [condition (2.1)]. We note that the solution written above corresponds to

$$C^l(x,0) = \frac{C_\infty(V - Kg)V\exp\{V\mathit{æ}^l x\}}{(V - Kg)\exp\{V\mathit{æ}^l x\} - B}.$$

In the limiting case $V = B + Kg$, where the solidification boundary is a characteristic, only the trivial solution $C^l = C^s \equiv 0$ is possible.

Remark 4.1. In the absence of microgravity $(g = 0)$, formulas (4.2) take the form

$$C^l(x,t) = \frac{C_\infty V\exp\{V\mathit{æ}^l x\}}{V\exp\{V\mathit{æ}^l x\} - B\exp\{V^2\mathit{æ}^l t\}}, \qquad C^s(x,t) = C_\infty$$

and give a solution of the problem in the case $V > B$.

Remark 4.2. We note that in the case $Kg + B \geqslant 0$, i.e., if the acceleration of microgravity is directed similarly to the temperature gradient, or it is directed in the opposite direction but the rate of transfer due to microgravity does not exceed in magnitude the rate due to the thermocapillary effect, the impurity is displaced into the liquid, i.e., $C^l(Vt,t) \geqslant C^s(Vt)$. In particular, the impurity is displaced into the liquid in the absence of microgravity.

In the case $Kg + B < 0$, i.e., if the acceleration of microgravity is opposite to the direction of the temperature gradient, and the rate of transfer due to microgravity is smaller in magnitude than the rate due to the thermocapillary effect, the impurity is displaced into the solid, i.e., $C^l(Vt,t) < C^s(Vt)$.

4.2. General Solution of the Transfer Problem in the Case where the Solution of the Stefan Problem is a Traveling Wave. Let the solution of problem (1.4) have the form (4.1). Then, the general solution of Eq. (1.6) in the case $V \geqslant B + Kg$ [condition (2.1)] is given by

$$C^l(x,t) = \exp\{V\mathit{æ}^l(x - Kgt)\}F\left(\frac{-B + (V - Kg)\exp\{V\mathit{æ}^l(x - Vt)\}}{\exp\{V\mathit{æ}^l(Kg - V)t\}}\right), \qquad x > Vt,$$

where $F(z)$ is an arbitrary smooth function for $z > V - B - Kg$, determined in the direct problem from the initial conditions $C^l(x,0) = C_0^l(x) > 0$ $[x > s_m(0)]$ by the formula

$$F(z) = \frac{V - Kg}{z + B} C_0^l\left(\frac{1}{V\mathit{æ}^l}\ln\frac{z + B}{V - Kg}\right).$$

In this case, the distribution of the impurity in the solidified part is obtained from conditions (1.7) and (1.8): $C^s(x) = C^l(x, x/V)(1 - (B + Kg)/V)$. In the case of the inverse problem, $F(z)$ is determined from the condition $C^s(x) = C_0^s(x)$ by the formula

$$F(z) = \frac{V}{z} C^s\left(\frac{1}{(V - Kg)\mathit{æ}^l}\ln\frac{z}{V - Kg - B}\right).$$

Remark 4.2 of the relation between the concentrations on the sides of the solidified and liquid regions at the boundary is valid in this case too. Figure 2 shows the concentration profile for the solid (region $0 < x < 2$ cm) and liquid (region $x > 2$ cm) regions at a time $t = 2$ sec for an Al–Pb emulsion with an
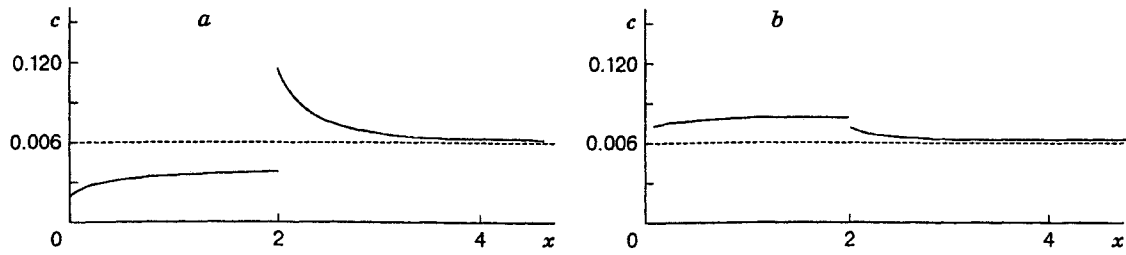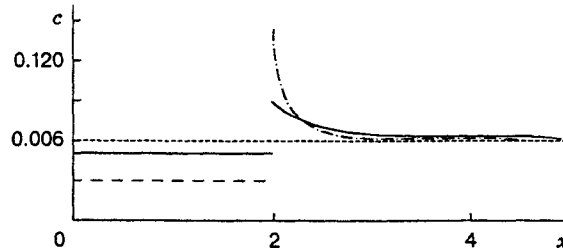
Fig. 2

Fig. 3

initial profile $C_0 = 0.006$ under the conditions of displacement of the impurity into the liquid (Fig. 2a) and accumulation of the impurity by the solid (Fig. 2b).

In the limiting case $V = Kg + B$, the concentration in the solid phase $C^s(x) = 0$, and the solution $C^l(x,t)$ with constant initial distribution $C^l(x,0) = C_0^l$ $[x > s_m(0)]$ increases without limit at the boundary: $C^l(Vt,t) = C_0^l \exp\{V \varkappa^l Bt\}$. This can be treated as the linearization effect; in the same limiting case for the nonlinearized problem, the concentration in the liquid approaches unity without bound.

4.3. Self-Similar Solution. Let $g = 0$. Then, using the standard self-similar solution of Stefan problem (1.4) (written, for example, in [3]) with the conditions $T^l(\infty,t) = T^\infty$ and $T^s(0,t) = T^0$, we obtain a self-similar solution of problem (1.6)–(1.8) that satisfies the condition $C^l \to C_\infty$ as $x \to \infty$:

$$C^l = C_\infty \exp\left\{ -\int_\xi^\infty \left(\frac{LT_{\xi\xi}^l}{\xi\beta^2/2 - LT_\xi^l}\right) d\xi \right\} \quad (\xi \geqslant 1). \tag{4.3}$$

Here $\xi = x/(\beta\sqrt{t})$ is a self-similar variable and β is a root of the equation

$$\gamma\beta^2/2 = k_m^s \frac{\partial T^s}{\partial \xi}(1) - k_m^l \frac{\partial T^l}{\partial \xi}(1).$$

Formula (4.3) is meaningful only in the case of rigorous positiveness of the denominator of the integrand for $\xi \geqslant 1$. For this, apparently, it suffices to require satisfaction of the inequality $\beta^2/2 > LT_\xi^l(1)$, which corresponds to condition (2.1) and can be expressed in terms of the input data. The maximum value of C^l is reached at the boundary $\xi = 1$:

$$C^l(1) = C_\infty \exp\left\{ -\int_1^\infty \left(\frac{LT_{\xi\xi}^l}{\xi\beta^2/2 - LT_\xi^l}\right) d\xi \right\}.$$

In this case, $C^s \equiv \text{const} = C^l(1)(1 - 2LT_\xi^l(1)/\beta^2) < C^l(1)$. Moreover, it is clear that $C^s < C_\infty$.

Figure 3 shows two concentration profiles — the solid and dot-and-dashed curves — for Al–Pb emulsion for various times; both profiles correspond to $C_\infty = 0.006$ and the position of the front $x = 2$ cm. The profile shown by the solid curve refers to the higher rate of solidification. In this case, the degree of purification of the mixture increases and the concentration at the boundary on the side of the liquid increases substantially.

4.4. Solution of the Transfer Problem with Self-Similar Temperature. First of all, we note that if the constraint $g = 0$ is rejected, by virtue of (2.1), a necessary condition for the existence of a nontrivial solution of problem (1.6)–(1.8) with self-similar temperature takes the form

$$\beta^2/2 > LT_\xi^l(1) + Kg\beta\sqrt{t}. \tag{4.4}$$

We consider following cases:

1. $g > 0$. Apparently, condition (4.4) cannot be satisfied for arbitrary values of time t, i.e., C^l can be only zero for $x \geqslant \beta\sqrt{t^*}$, where $t^* = (\beta^2/2 - LT_\xi^l(1))^2/(Kg\beta)^2$.

2. $g < 0$. Condition (4.4) is satisfied for $\beta^2/2 \geqslant LT_\xi^l(1)$. If $\beta^2/2 < LT_\xi^l(1)$, then, beginning with time $t^* = (\beta^2/2 - LT_\xi^l(1))^2/(Kg\beta)^2$, condition (4.4) is satisfied but the problem of the existence of a classical solution requires additional examination.

If microgravity is absent $(g = 0)$, the general solution of problem (1.6)–(1.8) with self-similar temperature is written explicitly, in a similar manner as is done in Sec. 4.2. Also, under the corresponding conditions, it is not difficult to write solutions of the direct and inverse problems.

4.5. General Solution of the Two-Front Problem in the Case where the Solution of the Stefan Problem is a Traveling Wave. Let, in problem (1.3), $T^l(x,t) = A^l(1 - \exp\{-V\text{æ}^l(x - Vt)\}) + T^d$, $B = LVA^l\text{æ}^l > 0$, $V > B + Kg$, $s_d(t) = Vt$, and $s_m(t) = Vt - Vt_0$, i.e., we assume that the matrix begins to solidify at time $t_0 = (V^2\text{æ}^l)^{-1}\ln(1 + (T^d - T^m)/A^l)$. We write a solution that satisfies the initial condition $C^l(x,0) = C_0^l \equiv \text{const}\ (x > 0)$. Then, we have

$$C^s(x) = \frac{(V - B - Kg)C_0^l(V - Kg)}{(-B + V - Kg + B\exp\{-V\text{æ}^l(x - Kgt_0)\})V}, \qquad 0 < x < Vt - Vt_0,$$

$$C^{sl}(x,t) = \frac{(V - B - Kg)C_0^l}{-B + V - Kg + B\exp\{-V\text{æ}^l(x - Kgt)\}}, \qquad \max\{0, Vt - Vt_0\} < x < Vt,$$

$$C^l(x,t) = \frac{(V - Kg)C_0^l\exp\{V\text{æ}^l x\}}{-B\exp\{V^2\text{æ}^l t\} + (V - Kg)\exp\{V\text{æ}^l x\} + B\exp\{V\text{æ}^l Kgt\}}, \qquad x > Vt.$$

We note that the presence of the second line of the concentration discontinuity and the intermediate layer, which is a liquid matrix with solidified inclusions, has little effect on the qualitative behavior of the solution for the liquid emulsion and completely solidified region.

REFERENCES

1. V. V. Pukhnachov and O. V. Voinov, "Mathematical model of the motion of an emulsion under the effect of thermocapillary forces and microacceleration," in: *Abstracts of the Ninth European Symposium on Gravity Dependent Phenomena in Physical Sciences*, Berlin (1995), pp. 32–33.

2. A. G. Petrova, "Monotonicity of the free boundary in the Stefan two-phase problem," *Dynamics of Continuous Media* (collected scientific papers) [in Russian], Novosibirsk, **67**, 97–99 (1984).

3. A. N. Tikhonov and A. A. Samarskii, *Equations of Mathematical Physics* [in Russian], Nauka, Moscow (1972).